1. In an arithmetic series

- the first term is 16
- the 21st term is 24
- (a) Find the common difference of the series.

(2)

(b) Hence find the sum of the first 500 terms of the series.

(2)

a)
$$u_1 = \alpha = 16$$
 $u_{21} = \alpha + 20d = 24$ $0 \leftarrow u_n = \alpha + (n-1)d$

$$16 + 20d = 24$$

$$20d = 8$$

$$d = \frac{8}{20} = 0.4$$

6)
$$S_{500} = \frac{500}{2} \left[2(16) + (500 - 1)0.4 \right]$$

= 57,900 ()
$$S_n = \frac{1}{2} n \left[2a + (n-1)d \right]$$

2. (i) In an arithmetic series, the first term is a and the common difference is d.

Show that

$$S_n = \frac{n}{2} \left[2a + (n-1)d \right]$$
 (3)

(ii) James saves money over a number of weeks to buy a printer that costs £64

He saves £10 in week 1, £9.20 in week 2, £8.40 in week 3 and so on, so that the weekly amounts he saves form an arithmetic sequence.

Given that James takes n weeks to save exactly £64

(a) show that

$$n^2 - 26n + 160 = 0 (2)$$

(b) Solve the equation

$$n^2 - 26n + 160 = 0 ag{1}$$

(c) Hence state the number of weeks James takes to save enough money to buy the printer, giving a brief reason for your answer.

(1)

(1)
$$S = a + (a+d) + (a+d+d) + ... + (a + (n-1)d)$$

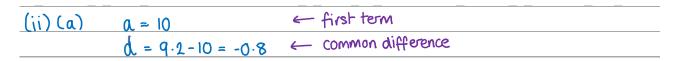
| reverse order of terms

$$S = (a + (n-1)d) + (a + (n-2)d) + ... + a$$
| add sequences by adding pairs of terms in each position

$$2S = (a + a + (n-1)d) + (a+2d+a+(n-2)d) ...$$
| for reversing + adding 1S = 2a + (n-1)d + 2a+(n-1)d ...

$$2S = n \times (2a + (n-1)d)$$

$$S = \frac{1}{2} n (2a + (n-1)d)$$
 as required



$$64 = \frac{n}{2} (2 \times 10 + (n-1) \times -0.8)$$

$$128 = n(20 - 0.8n + 0.8)$$

$$128 = 20.8 \, \text{n} - 0.8 \, \text{n}^2$$

$$0.8n^2 - 20.8n + 128 = 0$$
 $\div 0.8$

$$n^2 - 26n + 160 = 0$$
 as required.

(b)
$$(n-10)(n-16) = 0$$
 or use calculator / quadratic equation $\therefore n = 10$ and $n = 16$ 0

(c)	10 weeks - by 10 weeks he will have saved enough money.	
	10 weeks - by 10 weeks he will have saved enough money, so he wouldn't need to sove for b more weeks.	_

3. A sequence of terms a_1, a_2, a_3, \dots is defined by

$$a_1 = 3$$
$$a_{n+1} = 8 - a_n$$

- (a) (i) Show that this sequence is periodic.
 - (ii) State the order of this periodic sequence.

(2)

(b) Find the value of

$$\sum_{n=1}^{85} a_n$$

(2)

a) (i)
$$d_1 : 3$$

 $d_2 : 8 - 3 = 5$
 $d_3 : 8 - 5 = 3$ (7)
 $d_4 : 8 - 3 = 5$ (This sequence is periodic)

(ii) The order is 2 (1

b)
$$\geq a_n = 3 + 5 + 3 + 5 + \dots + 3$$

Total =
$$129 + 210 = 339$$

4. (a) Express $2 \cos \theta + 8 \sin \theta$ in the form $R \cos (\theta - \alpha)$, where R and α are constants,

$$R > 0$$
 and $0 < \alpha < \frac{\pi}{2}$

Give the exact value of R and give the value of α in radians to 3 decimal places.

(3)

The first three terms of an arithmetic sequence are

$$\cos x$$

$$\cos x + \sin x$$

$$\cos x + 2\sin x$$

$$x \neq n\pi$$

Given that S_9 represents the sum of the first 9 terms of this sequence as x varies,

- (b) (i) find the exact maximum value of S_{q}
 - (ii) deduce the smallest positive value of x at which this maximum value of S_9 occurs.

(3)

a)
$$2\cos\theta + 8\sin\theta = R\cos(\theta - \alpha)$$

$$R\cos \alpha = 2 \Rightarrow R = \sqrt{2^2 + 8^2} = 2\sqrt{17}$$

$$tana = \frac{8}{2} \Rightarrow \alpha = 1.326 \text{ rad } (3dp)$$

b) (i)
$$S_q = \frac{q}{2} (2a + (q-1)d)$$

$$\alpha = \cos x$$

(ii)
$$\cos(\Theta - 1.326) = 1$$

$$\theta = 1.326$$