1. In an arithmetic series

- the first term is 16
- the 21st term is 24

(a) Find the common difference of the series.

(2)

(b) Hence find the sum of the first 500 terms of the series.

(2)

a) $u_1 = a = 16$ $\quad u_{21} = a + 20d = 24$ ① $\leftarrow$ $u_n = a + (n-1)d$

$16 + 20d = 24$

$20d = 8$

$d = \frac{8}{20} = 0.4$ ①

b) $S_{500} = \frac{500}{2}\left[2(16) + (500-1)0.4\right]$ ①

$= 57,900$ ①

$S_n = \frac{1}{2}n\left[2a + (n-1)d\right]$

2. (i) In an arithmetic series, the first term is a and the common difference is d.

Show that

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

(3)

(ii) James saves money over a number of weeks to buy a printer that costs £64

He saves £10 in week 1, £9.20 in week 2, £8.40 in week 3 and so on, so that the weekly amounts he saves form an arithmetic sequence.

Given that James takes n weeks to save exactly £64

(a) show that

$$n^2 - 26n + 160 = 0$$

(2)

(b) Solve the equation

$$n^2 - 26n + 160 = 0$$

(1)

(c) Hence state the number of weeks James takes to save enough money to buy the printer, giving a brief reason for your answer.

(1)

(i) $S = a + (a+d) + (a+d+d) + \dots + (a+(n-1)d)$ 1

reverse order of terms

$S = (a+(n-1)d) + (a+(n-2)d) + \dots + a$

add sequences by adding pairs of terms in each position

$2S = (a+a+(n-1)d) + (a+2d+a+(n-2)d) \dots$ 1 for reversing + adding

$2S = 2a+(n-1)d + 2a+(n-1)d \dots$

$2S = n \times (2a+(n-1)d)$

$S = \frac{1}{2}n(2a+(n-1)d)$ 1 as required.

(ii) (a) $a = 10$ ← first term

$d = 9.2 - 10 = -0.8$ ← common difference

$64 = \frac{n}{2}(2 \times 10 + (n-1) \times -0.8)$ 1

$128 = n(20 - 0.8n + 0.8)$

$128 = 20.8n - 0.8n^2$

$0.8n^2 - 20.8n + 128 = 0$ $\div 0.8$

$n^2 - 26n + 160 = 0$ as required. 1

(b) $(n-10)(n-16) = 0$ ← or use calculator / quadratic equation

$\therefore n = 10$ and $n = 16$ 1

(c) 10 weeks – by 10 weeks he will have saved enough money, so he wouldn't need to save for 6 more weeks. 1

3. A sequence of terms $a_1, a_2, a_3, \ldots$ is defined by

$$a_1 = 3$$
$$a_{n+1} = 8 - a_n$$

(a) (i) Show that this sequence is periodic.

(ii) State the order of this periodic sequence.

(2)

(b) Find the value of

$$\sum_{n=1}^{85} a_n$$

(2)

a) (i) $a_1 = 3$

$a_2 = 8 - 3 = 5$

$a_3 = 8 - 5 = 3$ (1)

$a_4 = 8 - 3 = 5$ (This sequence is periodic)

(ii) The order is 2 (1)

b) $\sum_{n=1}^{85} a_n = 3 + 5 + 3 + 5 + \ldots\ldots + 3$

43 x 3's = 129 (1)

42 x 5's = 210

Total = 129 + 210 = 339

$\therefore \sum_{n=1}^{85} a_n = 339$ (1)

4. (a) Express $2\cos\theta + 8\sin\theta$ in the form $R\cos(\theta - \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < \frac{\pi}{2}$

Give the exact value of R and give the value of α in radians to 3 decimal places.

(3)

The first three terms of an arithmetic sequence are

$$\cos x \qquad \cos x + \sin x \qquad \cos x + 2\sin x \qquad x \neq n\pi$$

Given that S_9 represents the sum of the first 9 terms of this sequence as x varies,

(b) (i) find the exact maximum value of S_9

(ii) deduce the smallest positive value of x at which this maximum value of S_9 occurs.

(3)

a) $2\cos\theta + 8\sin\theta = R\cos(\theta - \alpha)$

$= R\cos\theta\cos\alpha + R\sin\theta\sin\alpha$

$R\cos\alpha = 2$ $\Rightarrow R = \sqrt{2^2 + 8^2} = 2\sqrt{17}$ (1)

$R\sin\alpha = 8$

$\tan\alpha = \frac{8}{2}$ (1) $\Rightarrow \alpha = 1.326$ rad (3dp) (1)

b) (i) $S_9 = \frac{9}{2}(2a + (9-1)d)$ $\quad a = \cos x$, $d = \sin x$

$= 4.5(2\cos x + 8\sin x)$

$= 4.5 \times 2\sqrt{17}\cos(\theta - 1.326)$

$\therefore$ max S_9 is when $\cos(\theta - 1.326) = 1$, and $S_9 = 4.5 \times 2\sqrt{17}$ (1)

max $S_9 = 9\sqrt{17}$ (1)

(ii) $\cos(\theta - 1.326) = 1$

$\theta - 1.326 = 0$

$\theta = 1.326$ (1)